A Scalable MVDR Beamforming Algorithm that is Linear in the Number of Antennas

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Motivation

- Large-scale antenna arrays are widely deployed in radar, sonar, wireless communication, and sensing applications
- Classical MVDR beamforming scales $O(M^3)$ due to covariance inversion
- Real-time implementation of MVDR beamforming in large arrays becomes computationally infeasible

Background

- Consider an array of M antennas receiving a desired signal and L interferers.
- The received signal at time t:

$$\mathbf{x_t} = \mathbf{a}(\theta_0)s_0(t) + \sum_{l=1}^{L} \mathbf{a}(\theta_l)\, s_l(t) + \mathbf{s_n}(\mathbf{t})$$

- The beamformer output:

$$\mathbf{y_t} = \mathbf{w^H x_t}$$

- The solution to MVDR Beamformer:

$$\mathbf{w} = \frac{R^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}(\theta_0)^{\mathbf{H}} R^{-1}\mathbf{a}(\theta_0)}$$

- Practically, R is not known and needs to estimated using received signal. R at time step n with m snapshots:

$$R_n = \frac{1}{m}\sum_{i=n-m+1}^{n} \mathbf{x_i x_i^H}$$

Proposed Method

- Recursive update rule for R:

$$R_{n+1} = \alpha R_n + (1-\alpha)\mathbf{x_n x_n}^{\mathbf{H}} (0 < \alpha < 1)$$

- Update for R^{-1} using Sherman-Morrison formula:

$$R_{n+1}^{-1} = \frac{R_n^{-1}}{\alpha} - \frac{(1-\alpha)}{\alpha}\frac{R_n^{-1}\mathbf{x_n x_n}^{\mathbf{H}}R_n^{-1}}{\alpha + (1-\alpha)\mathbf{x_n}^{\mathbf{H}}R_n^{-1}\mathbf{x_n}}$$

- Low rank SVD approximation:

$$R_{n+1}^{-1} \approx U_K D_{K,n+1}^{-1} U_K^H \quad (\mathrm{K} \ll \mathrm{M})$$

- Initialization: One-time low-rank SVD $O(M^3)$, thereafter, linear-scaling updates $O(MK^2)$

Algorithm

Inputs: Initial covariance matrix R_n, data vectors $\mathbf{x}_n, \dots, \mathbf{x}_N$, forgetting factor α, low-rank dimension K

1. Initialize: $R \leftarrow R_n$
2. Compute low-rank SVD: $R \approx U_K D_K U_K^H$
3. For each time step $i = n \;\dots N$:
4. $\quad \mathbf{x} \leftarrow \mathbf{x}_n$
5. $\quad$ Update the inverse submatrix:

$$D_{K,n+1}^{-1} = \frac{D_{K,n}^{-1}}{\alpha} - \frac{(1-\alpha)}{\alpha}\frac{D_{K,n}^{-1}U_K^H\mathbf{x_n x_n}^H U_K D_{K,n}^{-1}}{\alpha + (1-\alpha)\mathbf{x_n}^H U_K D_{K,n}^{-1} U_K^H \mathbf{x_n}}$$

6. $\quad$ Compute the Beamforming weights:

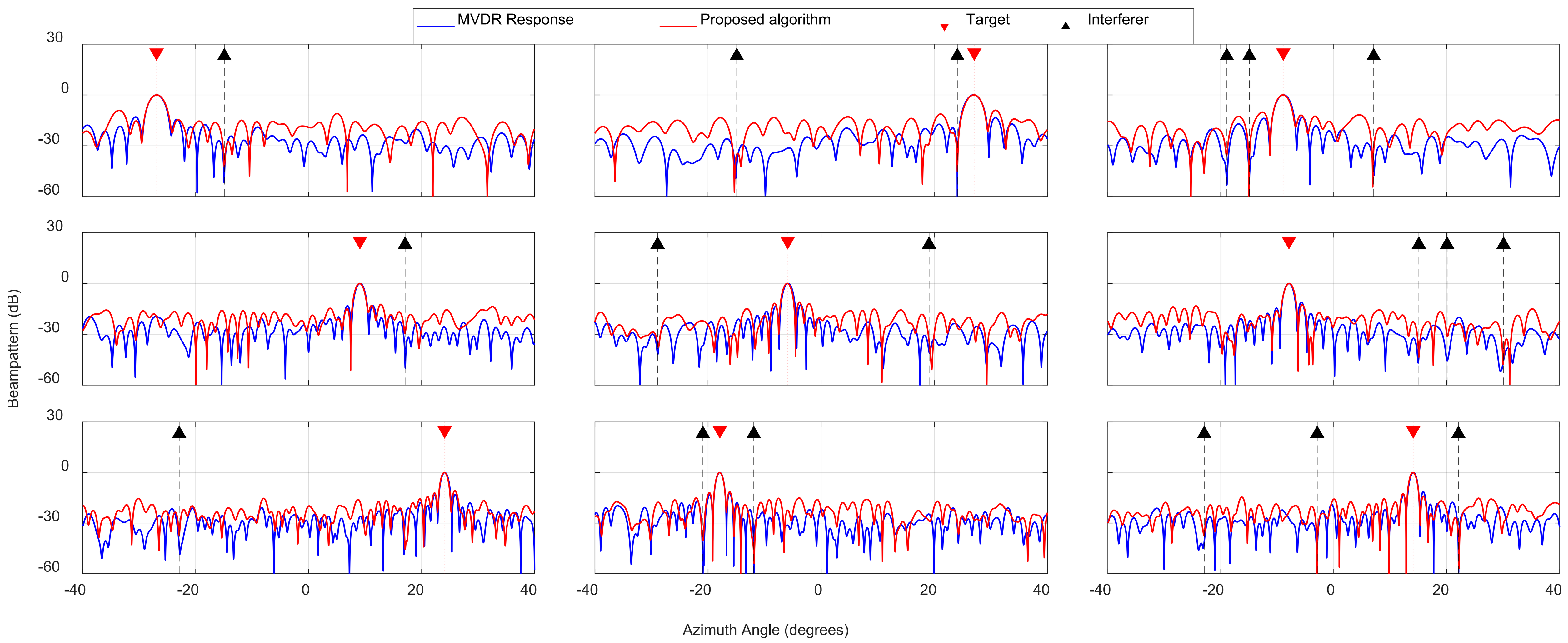
$$\mathbf{w} = \frac{U_K D_{K,n+1}^{-1} U_K^H \mathbf{a}(\theta_0)}{\mathbf{a}(\theta_0)^{\mathbf{H}} U_K D_{K,n+1}^{-1} U_K^H \mathbf{a}(\theta_0)}$$

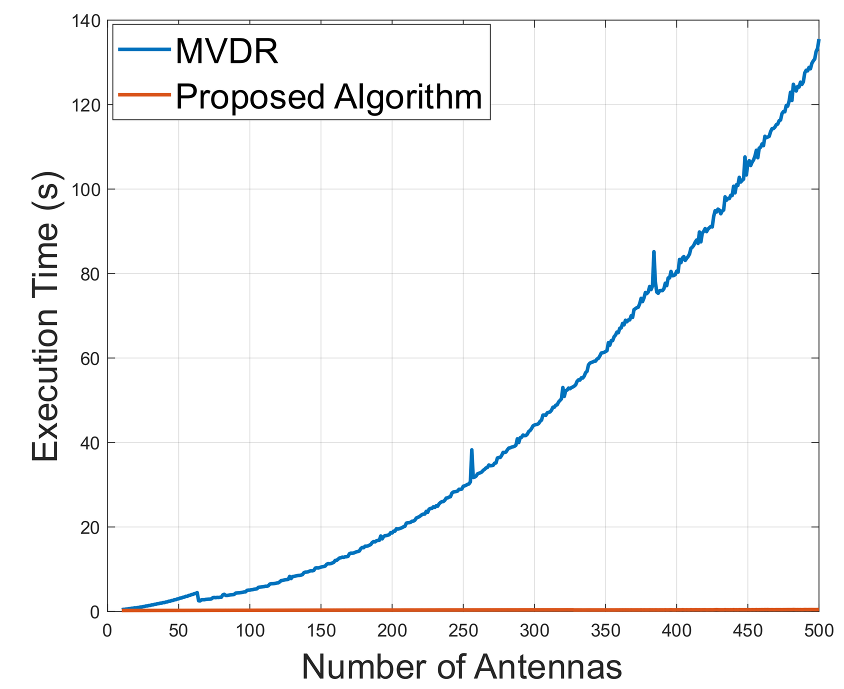
Output: Updated beamforming weights $\mathbf{w}$

Results

M	L	Main Lobe Width (°)↓		Null depth (dB)↓		Side Lobe Level (dB) ↓	
		MVDR	Proposed	MVDR	Proposed	MVDR	Proposed
50	1	2.27	2.32	−49.56	−42.44	−13.02	−9.14
50	2	2.38	2.31	−51.31	−40.10	−13.48	−9.06
50	3	2.09	2.03	−49.55	−35.29	−13.19	−10.20
75	1	1.37	1.37	−47.54	−29.32	−13.12	−12.32
75	2	1.36	1.33	−41.02	−34.84	−12.75	−11.05
75	3	1.34	1.41	−44.70	−39.53	−11.11	−12.55
100	1	1.10	1.08	−44.01	−37.13	−13.05	−10.76
100	2	1.07	1.07	−55.54	−44.56	−12.27	−11.47
100	3	1.06	1.08	−45.83	−41.78	−11.37	−12.47

Results





MVDR: $O(M^3)$

Proposed: $O(MK^2)$

Limitations

- When the signal is above the noise floor, the proposed method fails to place nulls as accurately as conventional MVDR
- The SINR gain gradually declines over time, but can be restored through periodic reinitialization